

THE CONSISTENCY OF INCONSISTENCY: ALAIN BADIOU AND THE LIMITS OF MATHEMATICAL ONTOLOGY

Tzuchien Tho (University of Georgia, USA)

Alain Badiou's reception in the English-speaking world has centred on his project of a "mathematical ontology" undertaken in Being and Event. Its reception has raised serious concerns about how mathematics could be relevant to concrete situations. Caution must be taken in applying mathematics to concrete situations and, without making explicit the equivocal senses of "consistency" as it operates in Badiou's thought, this caution cannot be precisely applied. By examining Being and Event as well as looking backwards at his first philosophical work, The Concept of Model, some key distinctions on the meaning of "consistency" will be clarified.

1. Ontology and Representation

Alain Badiou's project of "mathematical ontology" has constituted the centre of his work since he proclaimed in a 1984 review of J.-F. Lyotard's *Le Différend* in *Critique* that "*les mathématiques, dans leur histoire, sont la science de l'être en tant qu'être, c'est à dire de l'être en tant qu'il n'est pas, la science de la présentation imprésentable. Je le prouverai un jour.*"¹ Indeed, to date, it might be said that Badiou has made not merely one effort at doing so but two, in the first and second volumes of *Being and Event*. In the review Badiou reproached Lyotard for his reduction of mathematics to logic. He puts forth a key claim: "*je pose que le genre mathématique n'est sûrement pas réductible au logique, au sens où il est dit de ce dernier que si une proposition est né-*

¹ "Mathematics, in its history, is the science of being qua being, that is to say, being insofar as it is not, the science of the presentation of the unrepresentable. I will prove this one day." [my translation] Alain Badiou, "*Custos Quid Noctis,*" *Critique* 450, Nov. 1984, 861.

cessaire, elle n'a pas du sens."² Much of what forms the basis of Badiou's mathematical ontology has been staked on the appropriate distinction between logic and mathematics, a distinction cursorily raised in the review as a distinction between logical necessity and the sense of mathematics. If we follow the identification of mathematics with ontology (mathematics=ontology), what must be shown is the irreducibility of mathematics to logic. It is this identification that sustains the distinction made between being as such (*l'être en tant qu'être*) and being (*étant*) insofar as ontology would be the discourse of "being insofar as it is not." The two volumes of *Being and Event* are roughly split along these lines. Whereas the first volume deals with pure being in set theoretical formalisation, the second deals with being as it appears (or exists) treated by categorical formalisation (what Badiou calls "logic"). Yet, the strong distinction between being as such and existences provokes its own questions. In the recent reception of Badiou in the English-speaking world, commentators such as Peter Hallward, Ray Brassier and Oliver Feltham have frequently raised issues concerning the precise sense in which mathematical ontology relates to the world.³ How does mathematical ontology relate to concrete situations? What do the complex parts of set theory, for instance, have to do with the political action that took place last week?

Without jumping too quickly into our criticisms, let us properly lay out our problem since it is not entirely obvious what it is. We can turn to a treatment of this question in *Being and Event*.

Set theory, considered as an adequate thinking of the pure multiple, or of presentation of presentation, formalises any situation whatsoever insofar as it reflects the latter's being as such; that is, the multiple of multiples which makes up any presentation. If, within this framework, one wants to formalise a particular situa-

² "I posit that the mathematical is surely not reducible to logic, in the sense where it is said of the latter that if a proposition is necessary, then it has no meaning. [my translation]" *Ibid.*, 861.

³ See Peter Hallward, *Badiou: A Subject to Truth* (Minneapolis and London, University of Minnesota Press, 2003), 14. Oliver Feltham, "Materialism versus Empiricism: Differentiation, Dysfunction and Automata" (unpublished manuscript presented at the New School University, New York, November 2, 2006). Ray Brassier, "Presentation and Anti-Phenomenon" in *Continental Philosophy Review* 39, 2006, 59–77.

tion, then it is best to consider a set such that its characteristics—which, in the last resort, are expressible in the logic of the sign of belonging alone, \in —are comparable to that of the structured presentation—the situation—in question.⁴

This passage is deceptively straightforward but far from simple. We will not worry about a full explanation of the text. Suffice to say that, if being is irreducibly multiple, and set theory is adequate in formalizing this irreducible multiplicity, then this is its role in the formalisation of particular situations. That is to say, any given situation, piece of art, political action, etc., insofar as it is composed of a multiplicity of elements or the conjunction of multiple factors, can be formalised.

A brute application of set theory to reality no doubt falters on the necessary abstraction that it must engender in the analysis of things. A political situation no doubt concerns a multiplicity of factors, just as an artistic situation draws from a range of elements that situate its meaning. But, to read mathematics directly onto these diverse fields sterilises what is “political” or “artistic” about them. This is not to mention the oddness of the idea that mathematics might be relevant in the formalisation of these fields. Politics and art can indeed be considered in a formal way without any direct mathematical implications. Badiou certainly acknowledges this in his reading of the relation between mathematics and politics as a metaphorical one.⁵ This is what is recommended in Meditation Nine of *Being and Event*, concerning the political and ontological uses of the word “state.” We should then be suspicious of transplanting set theory or ontology directly onto concrete situations. Mathematics is not to be brandished about in a careless manner. Yet, the gap between mathematics and its supposed formalisation of particular situations raised in the citation above lacks a satisfactory answer. How does a set theoretical solution come to formalise concrete situations?

⁴ Alain Badiou, *Being and Event*, tr. O. Feltham (London and New York: Continuum, 2005), 130.

⁵ “Due to a metaphorical affinity with politics that will be explained in Meditation Nine, I will hereafter term state of the situation that by means of which the structure of a situation—of any structure whatsoever—is counted as one....” *Ibid.*, 95.

2. The Ontological Dialectic

Outside of a merely metaphorical use, the temptation to correlate sets with things and individuals or inclusion as political or artistic “representation” would disfigure the ontological nature of mathematics. Mathematical ontology would then function as the representation of particular situations. It would then be adequate when it “convincingly” analogises the stakes given in a particular situation according to some standard of representation. This would make “ontology,” if one can still call it that, subservient to some prior concept of what is at stake in analysis, whether it may be political, artistic or what concerns the anatomy of my cat.

Ontology’s turn toward representation is an easy temptation to forgive. After all, if ontology does not represent objects and their relations, then what does it do? It seems that a lot rests on this test of our interpretive acuity in reading the passage quoted above lodged in the first half of *Being and Event*. If what we encounter in reality are multiples within multiples, then sets seem to give us a nice representation of what things are and how they are related to one another. Giving in to this temptation, however, would result in the disjunction between mathematics and ontology. In this case it seems that we might fare just as well in the investigation of being by using the resources of natural language or phenomenology.⁶ The maintenance of the identity between mathematics and ontology would depend on severing ontology from representation. The science of being must be itself the same activity as mathematical thought. As such, set theory cannot be a formalised field of representations ranging over particular situations. Rather, it must, if one is to take Badiou seriously, constitute a field where being and its structure are actively thought.⁷

⁶ As I will discuss later in this article, Badiou does indeed take up the question of phenomenology in *Logiques des mondes*. It is important to stress that this project however does not investigate being as such but rather being as existence or appearance. Put in terms of the passage in *Critique* cited above, *mutatis mutandis*, this phenomenological approach would be an investigation of being insofar as *it is*, the science of the presentation of the *presentable*.

⁷ A quick glance at the “dictionary” appendix of *Being and Event*, will show that Badiou precisely defines what he means by “presentation” and “representation” (pages 519 and 521 respectively). Yet, my argument at this stage requires only a commonplace understanding of these terms. Representation is the presentation of something in terms other

Let us then exclude the interpretation of mathematical ontology given above. If mathematical ontology is not representational, then what is it? What does it mean for ontology to be the investigation of being as such outside of the relation between representation and the represented? Eschewing a representational approach, what we can assume, in the following investigation, are two theses that provide a means to resolve this question. The first is the thesis that being is irreducibly multiple; a “pure” multiple. This stems from our recognition that being does not provide its own means of unity, all unity comes from without. What this means, most importantly, is the indifference of being to ontology, that is, being’s independence from the differences (the distinctions that ontology designates) that constitute ontology. Hence, being as such is inconsistent in the sense that it is indifferent even to the most fundamental of distinctions (a “this” and a “that”). Simply put, being’s indifference to distinction is also its indifference to unity. Unity is thus “other” to being and being cannot but be purely and irreducibly multiple. The second thesis is that ontology constitutes itself only by being consistent. This thesis determines ontology as a rational task and immediately excludes the contemplation of being as a mystical experience, a pure presence before which one stands, a being that is as ineffable as it is unintelligible.

These two theses form a dialectical couple which pierces through our question concerning representation. We have before us the problem of inconsistent being supposed to be investigated by a consistent discourse. We should begin by saying that there is no argument for ontology’s consistency that does not turn out to be ultimately circular (consistency is already implied in any defence of it). Yet, while we might reasonably accept consistency as a necessary function of discourse, it is not evident why this should apply to ontology, especially given being’s supposed *inconsistency*. To see if we can defuse the problem, we should look at where the following assumption leads us. What if being is immanently consistent? This pushes us toward our key problem. Being is not ontology. Under the assumption that being is immanently consistent, ontology would be a structure other than being which adequately de-

than itself. A flower represents appreciation; a senator represents the will of her constituency. The bruteness of the term “presentation” is difficult to get around. It is what “there is” or what is “present.” Naturally, it is presented in terms of itself. We can easily see that all representations are presentations but presentations are not necessarily representations. Alain Badiou, *Logiques des mondes* (Paris: Éditions du Seuil, 2006).

scribes the immanent consistency of being. In this case, ontology cannot avoid playing a representational role. The means by which this consistency of being would be unfolded would be precisely that which is other than itself. If the representational character of ontology is to be avoided, the immanence of being's consistency (where being would determine itself as consistent) would have to be ruled out. We should recognise that the price to be paid for the possibility of ontology is the estrangement of being from consistency. At the same time, if we hold consistency to be the guiding force of ontological discourse, how can it hope to present being as such, which is indifferent, inconsistent and irreducibly multiple? Ontology is possible only as a science of difference. Yet, if the difference that it investigates is not immanent to being then ontology risks becoming un-ontological, becoming a science of difference as such and not a science of being as such.

This ontological dialectic, which is animated by the difference between the inconsistency of being and the consistency of ontology, concerns the very kernel of what consistency is. How does inconsistent being come to be presented in a consistent way? We should be conscious of the fact that ontology is a retroactive practice; it never finds its own stability. The autonomous constitution of ontology would imply that ontology is representational or worse, that it is not "ontological" insofar as its autonomy would imply its autonomy from being. The term Badiou uses for the consistency of situations, "presentation" is close to our commonplace notion of experience and of objects—a presentation of existence. Yet, presentation is a much larger term which captures not simply the consistency of phenomena but also the consistency of scientific theories, political sequences and the like. All these have important links to experience, to be sure, but cannot be reduced to it. What concerns ontology is not the consistency of experience, which is certainly the constitution of a "unity" out of a multiple set of givens, or the tracing of the ontological genesis of beings into some essence. Both of these cases would force us to revisit the nature of representation and correspondence as the distinction between being and ontology. Against this path, ontology can only be the presentation of being itself. In this sense, ontology is thus *a* situation and not *the* situation. If one is to exclude ontology from being a generalisation of all that exists, then its form of consistency cannot be, from this perspective, the hierarchical summit of other situations.

What must be presumed for the possibility of ontology is not the consistency of being, but that of presentation. This would be the consistency of ontology's own presentation. That is to say, ontology is possible as a particular and not as a general structure. If ontological consistency were to be the summation of what is consistent in presentation, then it would be the abstraction of the consistency of all situations. This abstraction would render ontological consistency a correlative image of the consistency of the world. Excluding this, only a particular situation can be called ontological. This particularity, what sets ontology apart from other presentations, is its presentation of being as such. With this, we have two senses of consistency. One sense concerns the consistency immanent to ontology, constituting its particularity. Another sense of consistency is extra-ontological. Insofar as two entities can be distinguished only against a consistent background of difference, ontology's difference from other situations can only be grasped according to a second sense of consistency.

3. Two Senses of Consistency

So far, we have not touched on the equivocation of consistency. I have been employing the word "consistency" in its loosely logical sense: a set of propositions is consistent insofar as it does not lead to a contradiction.⁸ In this sense, presentation is consistent insofar as it exhibits basic laws like that of the principle of non-contradiction, where something cannot be and not be at the same time and in the same respect. Consistency can also mean something with respect to its substance or constitution. This is the sense in which we ask about what something consists of. The French "*consistance*" and "*consistant*" tend to signify this second sense. When Badiou speaks of being as an inconsistent multiplicity does he mean an inconsistency that involves contradiction or simply that it

⁸ A more precise and general definition of consistency can be found in Badiou's *The Concept of Model*. Zachary Luke Fraser gives a brief résumé of this definition in his introduction. He writes, "The concept of logical consistency that Badiou adopts is informed by the definition formulated by the great American logician, Emil Post.... Post identifies a system's consistency with the existence of a statement that cannot be derived from that system, a statement whose derivation is impossible." Zachary Luke Fraser, "Introduction" in *The Concept of Model*, by Alain Badiou, tr. and ed. Z. L. Fraser and T. Tho (Melbourne: RE. Press, 2007), xxxii–xxxiii.

lacks constitution? Likewise, when presentation is said to be consistent, does that imply its ordered, rule-governed nature?

I turn to a passage in *Being and Event* where Badiou quotes G. Cantor on the question of inconsistent multiplicity. “[A] multiplicity may be such that the affirmation according to which *all its elements ‘are together’ leads to a contradiction*, such that it is impossible to conceive the multiplicity as a unity, as a ‘finite thing.’ These multiplicities, I name them *absolutely infinite multiplicities, or inconsistent...*”⁹ This is Cantor’s definition of an infinite multiple, a multiple whose unification, its consistency in the substantial sense, leads to a contradiction. This would be the logical sense in which irreducible multiplicity encounters a contradiction. It does not, however, mean that the pure multiple (in Badiou’s usage) is inconsistent in the logical sense. I will leave this double sense of consistency open for the moment. This equivocal sense of the term should, however, be kept in mind.

This issue of the double meaning of consistency structures much of Badiou’s thought on the nature of ontology. It is particularly striking in the development of his fundamental arguments concerning ontology’s consistent presentation of inconsistency. How is inconsistency to be consistently presented? The dialectic of ontology brings us to the puzzle of bridging the gap between inconsistent multiplicity and ontology as a consistent presentation. We took a first step by recognizing that ontology is neither the representation of the “hidden” structure in beings nor a generalisation of what is common in existing things. Instead, ontology can only be a presentation of the inconsistency of being as such. We can complete this operation in two more steps. First, every presentation is a multiple, but insofar as it is consistent, it is a multiple where unity is possible. This consistency of presentation is possible when a “this” is distinguishable from a “that,” a one and a two. Consistent presentation is entirely subsumed by the presentation of unity. In this sense, oneness or unity must be what is presumed in any presentation. Taking experience as an example, if we speak of an experience of an object, it is *an* object that is in turn composed of parts which can only be said to be parts insofar as they are counted as “one.” Citing Leibniz’s dictum, the proposition “What is not *a* being is not a *being*” is irrefutable if we are to maintain

⁹ *Being and Event*, 41.

the rigour of consistency in ontology.¹⁰ Yet, if existence can only be held for unities, then being as such, being irreducibly multiple, is unrepresentable. This unrepresentability of being does not intrude into consistent presentation as a “chaos” but as nothing; according to Leibniz’s formula, it is not a *thing*. This is the presentation of being “insofar as it is not,” the point where consistent discourse seizes upon the irreducibly multiple as a nothing or a void.

We should recognise that the void is not simply “nothing.” If that were the case, the void would be a representation of this nothing. Badiou’s alternative approach proposes that we take the void as a positive element in a consistent presentation, a thought that is nowhere more precisely actualised than in set theory. In Zermelo-Frankel set theory, the only set which is said to exist from the outset is the void set, the set to which no elements belong: $(\exists\beta)[\sim(\exists\alpha)(\alpha\in\beta)]$.¹¹ Out of the nine axioms of set theory, only two determine a positive existence in its consistent presentation, the axiom of the void set, and the axiom of infinity which nonetheless shares its consistency with the void set (an infinite set is the qualification of a set as a limit ordinal which is itself a set generated from the void set).¹² We should note that the axiom of the void set provides the most direct and transparent “presentation” of the void. The infinite multiple set out in the axiom of infinity is not “contradictory,” being different from Cantor’s “infinite” multiples cited above; rather, it is consistent insofar as it is conditioned by the void, which is what constitutes the first part of the axiom. In these two axioms, set theory is nothing but the presentation of this void, the consistent presentation of the unrepresentable.

Over the years, Badiou has provided many arguments for why set theory is identical to ontology; the tracing of these arguments is not my aim here.¹³ I wish to focus on how set theory is a theory of the void, a

¹⁰ *Ibid.*, 23.

¹¹ *Ibid.*, 68.

¹² The axiom of infinity states, $\exists x[\emptyset\in x\ \&\ \forall y(y\in x\rightarrow\exists z(z\in x\ \&\ \forall w(w\in z\leftrightarrow w\in y\ \vee w=y)))]$. There is a set which has \emptyset as an element and which is such that if a is an element of it then $\cup\{a, \{a\}$ or $(a\cup\{a\})$ is also an element of it. Mary Tiles, *The Philosophy of Set Theory* (Mineola, New York: Dover Publications, 2004), 122.

¹³ Outside of the line of argument that constitutes the first half of *Being and Event*, Badiou has presented basic versions of the argument for the identity between ontology and mathematics in the seven texts collected in the first section of *Theoretical Writings*, aptly entitled “ontology is mathematics.” These texts date from 1992 to 2000. Alain

presentation of the unrepresentable. Indeed this very characteristic is enough to identify ontology, which is the science of being as such, with set theory. The axioms of set theory are sufficient to provide the account of ontological difference. An example of this is the production of the natural numbers (1, 2, 3,...) by the difference between the void set and the set to which it belongs. Insofar as $\{\emptyset\}$ is not $\{\emptyset, \{\emptyset\}\}$, we can generate a sequence of sets upon which arithmetic operates. As such, \emptyset is zero, $\{\emptyset\}$ plays the role of one, $\{\emptyset, \{\emptyset\}\}$ is two and so on.¹⁴ This example provides only a bare appreciation of the ordered difference possible through the void set. While it demonstrates how a sequence of ordered presentations is possible starting from the presentation of the unrepresentable, one more step, however, needs to be taken in order to establish a basic framework of this theory of the void. We must see how set theory provides us with the identity between ontology and set theory.

The void set provides us with a means of a consistent presentation of the inconsistent multiple or the unrepresentable. What remains to be seen is how this seizure of inconsistent being gives way to some consequences which can establish the universe of sets as the field of ontological discourse. If the previous step accounts for how ontology is ontological (its thinking of being as such), our subsequent step should account for how ontology is consistent. Now, what is counted in set theory is the void set. If we take this counting to be on par with the set of natural counting numbers (one does not have to), then the null set \emptyset can be counted as zero, its count, the set $\{\emptyset\}$ is one, etc. This is just one of many ways in which set theory presents oneness as a result. Its most basic formulation is given as simple distinction. When at least one element of a set is not the element of another, then they are two different sets.¹⁵

Badiou, *Theoretical Writings*, ed. and tr. R. Brassier and A. Toscano (London and New York: Continuum, 2004), 3–93. One can also look to the less structured but highly informative and critical interview entitled “Ontology and Politics” at the end of *Infinite Thought*, conducted by Justin Clemens and Oliver Feltham in 1999. Alain Badiou, *Infinite Thought*, ed. and tr. O. Feltham and J. Clemens (London and New York: Continuum, 2004), 169–93.

¹⁴ Badiou provides just such a characterisation of the count in Meditation Seven of *Being and Event*, 92.

¹⁵ This is the precise sense of difference in ontology, given by the axiom of extensionality: $(\forall \gamma)[(\gamma \in \alpha) \leftrightarrow (\gamma \in \beta)] \rightarrow (\alpha = \beta)$. *Ibid.*, 61.

The unequivocal seizure of difference by the function of belonging \in is perhaps the defining characteristic of set theory. Accordingly, a set is not an "object" entangled in all its linguistic and epistemological complications, for it is simply defined by what belongs \in to it. This transparency, however, also implies the infinite variations possible in set theory. Sets α and β can be such that each has infinite members, none of which belong to the other. This infinite variation underlies the power of ontological presentation and its excess over predication, but how does ontology guarantee its consistency?

Taking any two sets, α and β , is it possible to show that what qualifies them as common is the void? Now, these two sets do not necessarily have to be the sets we discussed earlier in the construction of counting numbers; these two sets should be taken as two sets whatsoever. It is not apparent that the void has to be included in the two sets. Given that set α is {a,b,c} and β is {d,e,f} such that they do not share any common elements, it is apparent that the intersection of the two sets produce nothing in common. When we notice this fact, what we notice is that they include the void set in common such that the intersection between set α and set β is precisely the void set. Now clearly, the void does not belong to either of the two sets, but it is nonetheless included. Badiou provides an argument for this universal inclusion of the void in Meditation Seven of *Being and Event*. He begins not with any set theoretical notion, but with a logical one, the principle, "*ex falso quodlibet*", that $\sim A \rightarrow (A \rightarrow B)$. Given a contradiction, anything follows from the proposition. In arithmetical terms for example, If $x=4$ and $x=10$, then x can be anything whatsoever. Badiou's argument goes as follows: If no multiple (call it α) belongs to the void, then if α belongs to the void, then α belongs to any set whatsoever, $\sim(\alpha \in \emptyset) \rightarrow [\alpha \in \emptyset \rightarrow (\alpha \in \beta)]$. We can call this proposition P. According to the axiom of the void set, we hold that, α does not belong to the void (the antecedent of the proposition P). Given that the void contains nothing, α does not belong to it. If this is the case, then the consequent in the first conditional in P follows. Thus if α belongs to the void, then α belongs in all sets. Since the first segment of P, $\sim(\alpha \in \emptyset)$, holds absolutely by virtue of the axiom of the void, the rest holds absolutely. Thus, adding universal quantifiers, P gives us the fol-

lowing, $(\forall\alpha)(\forall\beta)[(\alpha\in\emptyset)\rightarrow(\alpha\in\beta)]$.¹⁶ Here, Badiou argues that, "since α and β are indeterminate free variables, I can make my formula universal: $(\forall\alpha)(\forall\beta)[(\alpha\in\emptyset)\rightarrow(\alpha\in\beta)]$. But what is $(\forall\alpha)(\forall\beta)[(\alpha\in\emptyset)\rightarrow(\alpha\in\beta)]$ if it is not the very definition of the relation of inclusion between \emptyset and β , the relation, $\emptyset\subset\beta$?"¹⁷ The sign \subset designates the relation between a subset of a set. Thus, even if the void set does not belong to a set, it forms part of its subset; the void set is included in every set. From this it follows that the intersection between two disjoint sets (sets with no elements in common) is the void set. In a way, it rearticulates the gap between the void set, and the count-as-one in that the set $\{\emptyset\}$ is not the void set, but the counting of it, the "one" distinguished from the zero of the set \emptyset . However, it also allows us to grasp something unseen. The theorem shows how the emptiness of the void set allows it to qualify as common between any two sets in the universe of sets: it is what they share by virtue of inclusion.

The universal inclusion of the void means that the intersection between two sets whatsoever is comparable with the void set. That is to say, there is no multiple that does not include within it some part of the "inconsistency" that it structures. The diversity of multiplicity can exhibit multiple modes of articulation, but as multiples, "[T]hey have nothing to do with one another, they are two absolutely heterogeneous presentations, and this is why this relation—of non-relation—can only be thought under the signifier of being (of the void), which indicates that the multiples in question have nothing in common apart from being multiples."¹⁸ The universal inclusion of the void thus guarantees the consistency of the infinite multiplicities immanent to its presentation. That is to say, it underlines the universal distribution of the ontological structure seized at the point of the axiom of the void set. The void does not merely constitute a consistency at a local point but also organises, from this point of difference, a universal structure that legislates on the structure of all sets, the universe of consistent multiplicity.

This final step, the carrying over of the void seized as a local point of the presentation of the unrepresentable, to a global field of sets provides us with the universal point of difference, applicable equally to

¹⁶ *Ibid.*, 87.

¹⁷ *Ibid.*

¹⁸ *Ibid.*, 186.

any number of sets, that guarantees the universal consistency of ontological presentation. While our argument concerning the axiom of the null set establishes set theory as a theory of the void, thereby identifying ontology as ontological, this second step establishes the universal consistency of ontology. Yet, keeping in mind our worry of the equivocal nature of the term “consistency,” what sort of consistency is now being presented in ontological discourse?

In one sense, the universal inclusion of the void demonstrates that, as a unit of presentation, the void anchors the set theoretical universe by its universal inclusion. As such, every presentation in ontological thought is situated in this elementary seizure of ontological difference. The void is that which “fills” ontological or set theoretical presentation. It is what makes common the universe of sets. It is in this sense that the “substance” or constitution of ontology is the void. At the same stroke, however, the universal inclusion of the void also concerns the consistency of set theory in a logical sense.

The universal inclusion of the void provides an important synthesis of the consistency of presentation. What is presented is necessarily consistent but its consistency gives way to two distinct senses. Consistency can refer to its own “substance,” its immanent presentation. Distinct presentations constitute different presentations principally because “what” they present are different. Ontology’s particularity is its presentation of the void. On the other hand, a political site might present certain elements just as a scientific procedure might present yet others. The other sense of consistency is tied to presentation as such, the consistency of presentation in its generality. When one speaks loosely about the “world” being consistent, where natural laws are verifiable against a background of regularity, it is this consistency that is invoked and not the elements that constitute the particularity of their presentation. This sense of consistency, occurring across presentations would certainly take us beyond the particularity of ontology. As such, in line with what we have been discussing so far, it would have to be analysable in multiple ways. That is to say, ontological presentation presents a species of this consistency. However, the possibility of multiple approaches does not exclude an ontological treatment of this consistency.

Badiou treats the distinction between the two consistencies by distinguishing between structure and meta-structure, arguing that all presentation is “structured twice.” He remarks that, “All I am saying is

this: it is on the basis of chaos not being the form of the donation of being that one is obliged to think that there is a reduplication of the count-as-one.”¹⁹ The consistency of presentation in its generality expresses what is given in Leibniz’s dictum, namely, that what is must necessarily be one. With this, we can return to see what the universal inclusion of the void gives us. As I mentioned, the consistency of presentation as such should admit multiple forms of analysis, but since ontology is (a) consistent presentation, this general feature can also be analysed along ontological means. The universal inclusion of the void allows us to determine a general feature of presentation. Its general statement, $\emptyset \subset \beta$, allows us to determine that given two disjoint multiples, α and β , there is a multiple that is included but does not belong to either set. That is to say, when all the elements of a situation are enumerated, there is always some multiple (included but not belonging) that escapes the count.

We should note, however, that this positive ontological result, the excess of inclusion over belonging, can be effectively carried over into presentation as such, regardless of its proper consistency. What gives this ontological result its uncharacteristic generality? Ontology is not only the consistent presentation of inconsistency; it is also a consistent presentation. This is the distinction between what is being presented and the presentation itself. Thus when a theorem of ontology, like that of universal inclusion, achieves some precision on a structural or universal level, it is pronounced on the nature of presentation itself, on the condition of presentation’s consistency. It thus allows itself to be transposed into other regions of presentation, insofar as they are consistent. Despite being thought in an ontological domain, its consequences can apply extra-ontologically. Thus, just as ontology is not the only domain of thought, it is not the only domain of thinking multiplicity. If ontology is the particular actualisation of a form of thinking multiple, the presentation of the unrepresentable, then its results surely reflect back onto the wider context.

We should not overlook the explicit logical background against which the theorem of universal inclusion is derived. It draws from the logical principle, *ex falso quodlibet*, that an inconsistent multiplicity, in the sense that it entails contradictory propositions, implies any proposition whatsoever. This principle, sometimes termed the principle of “ex-

¹⁹ *Ibid.*, 94.

plosion” implies, across all presentation, that the consistency of a situation involves the sort of excess of multiplicity derived in the theorem of universal inclusion of the void.

4. The Logic of Situations

If we were to play a language game, one Badiou himself plays in the preface of *Logiques des mondes*, it might go something like the following.²⁰ Ontology is split between two components. The first being “*ontos*” and the second being “*logos*.” Where ontology concerns being as such, its consistency concerns the “*ontos*” of ontology, the literal sense of what is being presented in ontology. Where ontology concerns consistency, it is a question of the “*logos*” of presentation, the sense in which ontology is logically relevant. The first part would concern ontology’s immanent conditions, the second, insofar as it concerns consistency in the logical sense, will concern the presentation as such. It is this second characteristic that allows for ontology’s extra-ontological relevance and applicability. In what follows I hope to solidify this reading. Yet, without entering too quickly into this final part of interpretation, I hope to reconsider what is at stake.

The theorem of universal inclusion demonstrates a number of interesting features of Badiou’s ontology that have not been made explicit. In fact, it stands as a sort of universal proposition that not only structures ontology’s immanent consistency but also allows it to be carried into other presentations. Allow me to reiterate that ontological presentation is neither a representational procedure nor a free-floating hermeneutic of pure difference. The universal inclusion of the void provides us with a scope of the ontological situation; it situates set theoretical presentation as nothing more and nothing less than a theory of the void. What this reinforces is ontology’s indifference to “concrete situations.” The differences immanent to ontology are generated by the consistent presentation of being as such. Insofar as this is the case, ontology’s differences, presented as the syntax surrounding the axiom of the void, do not carry over into any other domain. That is to say, if we are to be serious about ontology being a situation, then sets cannot be objects and its presentation cannot be an analogy for “the way things are.”

²⁰ Alain Badiou, *Logiques des mondes* (Paris: Éditions du Seuil, 2006), 48.

Certainly, the consequences of ontological investigation, just like consequences of artistic or political acts, can be carried over into other domains; whether as inspiration or as the opening up of common interstices, these can be as surprising in the way they provide solutions to previously irresolvable problems as they are problematic, insofar as they posit new problems. What is key, however, is to recognise that these innovations are not ontological. In this sense, ontology does not guarantee nor prescribe the consistency of any other situation. Nonetheless, in any situation, the terms of its consistency can be read according to the relation between the unrepresented, the presented, inclusion and exclusion. When it comes down to the analysis of these terms, the positive results of ontology appropriately intervene. The analysis of these points, then, does not concern the situation itself, but its consistency and inconsistency. We should be clear in saying that when the transformation engendered by an artistic movement or a political act is analysed according to its consistency, we lose any sense of its political or artistic nature; it would be rarefied abstraction.

Badiou makes this point clear when he remarks that “the investigation of any effective situation (any region of structured presentation), whether it be natural or historical, reveals the real operation of the second count. On this point, concrete analysis converges with the philosophical theme: all situations are structured twice.”²¹ This “second count” is a reference to the double structure of consistency (or structure) analysed in the previous section. The analysis of a concrete situation concerns the consistency of its presentation and not that of ontology’s. In other words, ontology conceived as an active field of presentation has a being of its own; it circumscribes its own situation via its proper form that answers to a particular dialectical field. This field, despite its particularity, displays its universality in the consistent formalisation of the inconsistent multiplicity at work in the presentation and representation operating in concrete situations. This level of analysis, despite being possible in any situation, is not always relevant to the effective distinctions and differences operating in these diverse situations. The carrying over of positive ontological results to other situations will then concern only the concrete situation’s mode of presentation. The difference presented and investi-

²¹ *Being and Event*, 94.

gated in ontology is thus indifferent, as it were, to what is being presented in those fields be it an artistic form or a scientific hypothesis.

In light of this, we can return to the passage quoted from *Being and Event* at the start of our investigation. What Badiou says is nothing other than this: the formalisation of a situation along ontological lines is the formalisation of its inconsistent multiplicity, its being as such. Nothing like representation is involved and the arcane application of set theory directly on things or experiences are rigorously excluded. The passage must be read as a claim about the multiplicity of presentation as such. Hence, the formalisation of any situation according to ontology proceeds along the lines of consistent multiplicity, that is, the consistency of presentation.

What made the passage difficult was the insufficient distinction between the two senses of consistency. When Badiou remarks that, “the sole term from which ontology’s compositions without concept weave themselves is necessarily the void...,” we should understand ontology’s consistent presentation in its “substantial” or constitutional sense.²² That is to say, ontological presentation owes its consistency to the mark of the void and the set theoretical syntax that actualises its intra-theoretical consistency. Likewise, when we read Badiou arguing that, “Plato thus formulates an essential ontological truth; that in the absence of any being of the one, the multiple in-consists in the presentation of a multiple of multiples without any foundational stopping point.”²³ This “in-consistency” is quite literal in the sense that irreducible multiplicity occupies what is being operated “on” in ontological presentation. This substantial sense of consistency highlights what is a central theme of Badiou’s mathematical ontology, i.e., its syntax, the very terms by which the form of ontological thought is put into practice. Yet, as we mentioned earlier, the other sense of consistency is equally important. Without a regime of logical consistency, there would be no theorems (since no proposition can be qualified as valid) and hence no consequences. Worse, ontology would fail to be ontological, for it would lack the resources to distinguish between two identities; a set α would be indistinguishable from set β without logical consistency since it would be impossible to distinguish whether they “contained” the same elements. Worse yet, with respect to the axiom of

²² *Ibid.*, 57.

²³ *Ibid.*, 33.

the null set, the operator “not” would be meaningless. Ontology as consistent presentation thus relies on the two senses of consistency: first, the constitutional sense of consistency allows that ontology be properly speaking ontological, it determines the seizure of being as such, which is inconsistent multiplicity; second, the logical sense of consistency is what drives the motor of ontological derivation.

How does this pose a problem? I think that it is naïve to object to Badiou’s ontology by saying that since ontology depends on a logical sense of consistency, governed by the rules of classical logic, then the identity between ontology and mathematics ceases to hold. Obviously, ontological presentation remains tied to a set theoretical syntax, and it is thus irreducible to classical logic. The presentation of the unrepresentable in set theory’s theorisation of the void is a capacity that escapes classical logic, despite requiring it as a background conception.

What is minimised, however, in Badiou’s presentation of ontology in *Being and Event*, is precisely the explicitly logical dimension which, without condemning his project to contradiction, does leave some fundamental questions unanswered. This is precisely what is lacking in a full explanation of the relation between ontology and concrete situations. What we said above about the carrying over of ontological conclusions to non-ontological situations is precisely what is at stake here. While the first sense of consistency codifies how ontology thinks its own presentation (the presentation of the unrepresentable), the second sense codifies how ontology thinks presentation as such.

Now, when I say that this second sense of consistency is minimised in *Being and Event*, I do not mean to say that Badiou has ignored it. Let us not forget that the context wherein Badiou first announced his project of mathematical ontology, in his review of Lyotard, was one where the distinction between logic and mathematics was the central concern. In closing, I propose that we cast a short glance backwards to Badiou’s first philosophical text, *The Concept of Model*. In making this short observation, I will clarify the sense of his extra-ontological reading of ontological consistency.

In *Concept of Model*, a key distinction that Badiou makes, anticipating his review of Lyotard’s text, is the distinction between logic and mathematics. Badiou argues that,

The surest criterion amounts to saying that an axiom is logical if it is valid for every structure and mathematical otherwise. A mathematical axiom, valid only for particular structures, marks its formal identity by excluding others through semantic force. Logic, reflected semantically, is the system of the structural as such; mathematics, as Bourbaki says, is the theory of the species of structure.²⁴

Here, Badiou argues that the major difference between mathematics and logic is its scope—the range over which its statements are valid. As such, the seizure of the mathematical comes only from the exclusion of some structures by means of “semantic force” or the range of statements it is valid for. In the context of *The Concept of Model*, this point is made with some short demonstrations concerning the interpretation of the validity of some first order logical propositions with respect to the universe of elements over which they range. While I will not enter into these demonstrations here, the central point concerns just how mathematics constitutes a “species of structure.” The difference that occasions this “speciation” occurs along a larger logical context that brings to the forefront the positive limits of a mathematical discourse.

Of course, not all logical models are mathematical and the limitation of scope in a logical formula does not immediately plunge us into set theory. Yet, reading forward into *Being and Event*, it is through such a limit that the *singularity* of the empty set is seized. This singularity does not follow from the dialectic of being, since the dialectic does not itself prescribe the mathematical form. But we should recognise that the void’s singularity is seized in the role that it plays against the background of consistent presentation in the logical sense. While *Being and Event* gives an account of ontological consistency by going in the direction of ontology to consistency, we can achieve some sense of how an account can function in the reverse, from consistency to ontology. Badiou writes in *The Concept of Model*:

[O]ne cannot have any rational discourse about logical principles (except to state their ‘evidence’), because rationality is precisely defined by the conformity of discourse to these principles. Logic

²⁴ *The Concept of Model*, 35.

would always already be there, consequently conditioning, rather than resulting from, the history of Reason. We are tempted to say that in reality, logic is itself an historical construction.... The 'circle' is resolved in the gap between demonstrative practice and experimental (or 'formal') inscription, the gap that is the motor of this science history. This mode of historical existence does not at all differentiate logic from mathematics.... Only mathematical axioms lift this semantical indistinction, and produce the effective inscription of a structural *gap*, by which the concept of model is legitimated.²⁵

It is in this light that Badiou's statement concerning mathematical ontology as a historically contingent situation (and its subsequent irreducibility to logic) is best read.²⁶ The syntactic consistency of ontology is an index of its historicity, yet this consistency is to be read against a background of semantic "indistinction," making distinct what revolves around its rational conditions. This is a feat that syntax by itself cannot accomplish. This historical index, which must be distinguished from the adequacy of set theory to present the unpresentable, is what brings ontology into relation with concrete situations in the way we have discussed, namely, the consistency of presentation. This would be consistency taken in its general sense, that is to say, consistency indifferent to the presentation of ontology. We are then in the position to distinguish two moments in ontology, a distinction that clarifies the singularity of set theory: consistency along syntactical or formal lines and consistency along logical lines. The syntactical consistency, following Badiou's meticulous demonstration in *Being and Event*, constitutes ontology's intrinsic and fundamental character as mathematical. Its logical consistency, what constitutes its difference from universal logical rules, is what provides ontology's capacity to think presentation's consistency, that is, its historicity.

Insofar as this holds, ontology's relation to concrete situations concerns the consistency of presentation. Against a backdrop of consistent presentation, the ontological situation seizes upon a singular point of entry, the void as the presentation of the unpresentable. This act designates ontology as a particular form in a background of consistent presen-

²⁵ *Ibid.*, 37.

²⁶ *Being and Event*, 14.

tation. Without this background consistency, the void would fail to express its singularity. As such, each instantiation of this act is particular insofar as the invention of a new artistic form or a form of political contestation can also be read as the historical production of a consistent form. But, what relates these as acts is the consistency that ranges across situations. It is in this sense that the positive result of ontological inquiry can be transposed to situations, or consistent presentation, when what is in question is the consistency of presentations. Thus, when this question of consistency is applied back onto ontology itself, it allows us to grasp the extra-ontological consistency of ontology. It allows us to recognise the specificity of ontology as a situation alongside others. This consistency is ahistorical when considered independently, that is, according to its own immanent difference; when it enters into the particularity of situations, however, what we achieve are the indices generated in the particularity of a new mode of production. Ontology's historicity is precisely the crucial link that both sets itself apart from other situations and, by the same stroke, puts it in a relation with them insofar as it renders consistency visible as an historical condition.

In a recent interview accompanying the recent translation of *The Concept of Model*, Badiou explains that he has come to regard the distinction between logic and mathematics given in Bourbaki's formulation as "insufficient," claiming that the purely formal distinction of "speciation" was no longer adequate to account for the difference between mathematical ontology and logic.²⁷ Yet, what is not disavowed is the way in which mathematics actualises the singularity of a form which is only so by its differentiation from and stratification of the given "always already" consistent background. It is this idea of an active thought tearing away from consistency that constitutes what Badiou regards as his "single idea." Badiou admits that, "I have a very simple and minimal idea, that all thought is the opening up of formalisation. Mathematics is only a particular kind of formalisation among all the various differentiations and complex productions which stand in the relation, on the one hand, to universality, the universality of form, and, on the other, to the particularity of its world."²⁸ This opening up of formalisation, mathematical or otherwise, is the actualisation of a particular against ahistorical

²⁷ *The Concept of Model*, 85.

²⁸ *Ibid.*, 103.

consistency. It concentrates, in each act, a particular form of experimentation and production that render it not only open to the contingency of its particular situation, but as the index of historicity.

5. Heterodox Platonism and Materialist Epistemology

We began with Badiou's statement in 1984 that he was going to prove the equivalence of ontology and mathematics. This proof, as he recognised then, concerned the distinction between logic and mathematics. This distinction however, in light of his ostensible "proof" in *Being and Event*, raised the question of how ontology could ever achieve any relation to actual concrete situations. Following this, we examined the fact that the consistency of mathematical ontology obliges us to recognise its non-representational character insofar as ontology does not represent nor establish a correlative relation with things and experiences in the world. As such, the non-representational character of ontology must follow a course where it is the presentation of the unrepresentable. In this, ontology achieves a particular status, its identity with set theoretical formalism. Yet, this particular status did not provide the means to fully explain its relation to other non-ontological situations. As such, we pointed to the equivocation inherent in the term "consistency." The examination of this equivocation allowed us to see two aspects of ontology, one concerning its proper and particular task of thinking being and a second, concerning the way in which this particularity reflects back onto a more general context of consistent presentation. By looking at the distinction between logic and mathematics made in one of Badiou's earliest philosophical texts, *The Concept of Model*, we saw that this double sense of consistency is indeed the very distinction between logic and mathematics. As such, mathematical ontology, apart from being non-representational, is to be seized as a particular mode of experimentation and production against the background of logical consistency. This means that the ahistorical modes of consistency find historical indices in mathematical production. This key idea of formalisation allowed us to grasp the logical consistency of being as constituting at the same time a positive relation with ontology, the science of being as such, and the logical consistency of presentation in general. Here, the relation between mathematics and concrete situations in Badiou's thought achieves a positive footing in the extra-

ontological consequences of ontology articulated against the background of the consistency of presentation as such.

A complete explanation of this central feature of formalisation in Badiou's thought would certainly have to include an investigation of the question of classical and non-classical logic in *Logiques des mondes*, where his reflections on logical forms are explicit. Yet, what I hope to have shown is the important though implicit role that logic has already played in *Being and Event*. This clarification allows us to thoroughly dismiss any representational relation between ontology and concrete situations. Rather, the relation that ontology establishes is the operation of concrete situations under the mode of logical consistency. This affords us with some perspective on how a non-representational mode of ontology might operate but it also shows how consistency as such might guide, without the means of correspondence, the understanding of the historicity of thought and, more importantly, the relation between mathematical ontology and concrete situations. It operates by the consistency of formal or syntactical invention in thought and at the same time provides positive results on the very meaning of logical consistency in presentation. We might call this Badiou's uncompromising but heterodox Platonism, where thought, guided by consistency, is actualised in the bold inscription of form. We might equally call this Badiou's epistemological materialism, where thought is only ever actual in the particular and active modes of production, disciplined by a form torn away from an indistinguishable and ahistorical background of consistency.

tzuchien.tho@gmail.com