Modality as a Subject for Science

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Abstract: Section 1 introduces the category of objective (non-epistemic) modality, closely related to linguists' category of circumstantial or dynamic modals, and explains metaphysical modality as its maximal element. Section 2 discusses various kinds of skepticism about modality, as in Hume and recent authors, and argues that it is ill-motivated to apply such skepticism to metaphysical modality but not to more restricted objective modalities, including nomic modality. Section 3 suggests that the role of counterfactual conditionals in applications of scientific theories involves an objective modal dimension. Section 4 briefly discusses the role of objective probabilities in scientific theories as exemplifying the scientific study of objective modality. Section 5 summarizes a case study of dynamical systems theory, widely used in natural science, as a mathematical theory whose intended applications are objectively modal, as perspicuously articulated in a language with modal and temporal operators and propositional quantification. State spaces in natural science characterize objective possibilities. Section 6 argues that, although those possibilities are usually more restricted than metaphysical possibility, their scientific study is a partial study of metaphysical possibility too.

Philosophers often discuss 'metaphysical' modality in isolation. Yet it is just a special case of a broad range of modalities, which we may call 'objective' to contrast them with epistemic, doxastic, deontic, and teleological modalities. Metaphysical possibility, physical possibility, and immediate practical possibility are all types of objective possibility. When we study the metaphysics and epistemology of metaphysical modality, we should do so as part of a broader study of the metaphysics and epistemology of the objective modalities, to avoid a distorted view of the matter. Since such modalities are in general open to natural scientific investigation, we should not treat the metaphysics and epistemology of metaphysical modality without reference to the metaphysics and epistemology of natural science.

The paper starts with a preliminary sketch of metaphysical modality and its place in the general category of objective modality, followed by a review

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of some familiar forms of skepticism about metaphysical modality in that light. Then a few of the many ways in which natural science concerns questions of objective modality will be explored.

1 Objective Modalities

Things could have been otherwise. That is not an epistemological point, for we know trivially that things are not otherwise, that they are whatever way they actually are, even if we do not know which way that is. The relevant modality is *objective*, in a sense contrasted with the epistemic and the doxastic, just as some probabilities (e.g., chances) are objective rather than epistemic or doxastic (e.g., credences).

Objective modalities are non-psychological and non-intentional. Thus they are not sensitive to the guises under which the objects, properties, relations, and states of affairs at issue are presented. Thus since it is objectively impossible for Hesperus to be distinct from Hesperus, it is objectively impossible for Hesperus to be distinct from Phosphorus. By contrast, epistemic modalities presumably *are* sensitive to guise. In prehistoric times, it was in some sense epistemically possible that Hesperus was distinct from Phosphorus. The matter is admittedly delicate, because on coarse-grained views of the individuation of propositions the proposition that Hesperus is Hesperus just *is* the proposition that Hesperus is Phosphorus, so the latter proposition is epistemically impossible if the former proposition is. Indeed, unless the context "the proposition that . . ." is quotational or the like, Leibniz's law requires that since Hesperus is Phosphorus and the proposition that Hesperus is Phosphorus is the proposition that Hesperus is Phosphorus, it is the proposition that Hesperus is Hesperus. But even such views have to make the relevant epistemic distinctions *somehow*. In some sense, the distinctness is epistemically impossible under the guise of the sentence "Hesperus is distinct from Hesperus" but not under the guise of the sentence "Hesperus is distinct from Phosphorus." Then the pertinent contrast is that objective modality admits no such non-trivial relativization to guises. The distinctness is simply objectively impossible irrespective of the guise under which it is presented.

In linguistics, the distinction between epistemic and non-epistemic modality is widely taken to be fundamental to the taxonomy of modal constructions in natural languages, such as almost omnipresent auxiliary verbs like 'can' and 'must.'[1] The present category of objective modality corresponds roughly to Angelika Kratzer's 'root' or 'circumstantial' modals and to Paul Portner's 'dynamic' modals. Although our present concern is primarily with the objective modalities themselves, rather than the semantic means

[1] See Kratzer 2012, 49–62 and Portner 2009, 144–184, for instance. It is not denied that the same word can express an objective modality in one context and an epistemic modality in another: compare "She could run a marathon in three hours" (objective) with "Goldbach's Conjecture could be true" (epistemic).

of expressing them in natural languages, the theoretical significance of the category gains some defeasible support from its apparently universal role in human thought and talk. Similarly, although the separation of objective probabilities from epistemic probabilities is primarily motivated by theoretical concerns in the philosophy of probability, its affinity with distinctions marked in natural languages gives further support to its significance (Kratzer 2012, 61). Of course, one might argue that the distinction between 'objective' and 'non-objective' modalities is less clear than it looks, but how many useful distinctions are perfectly clear?

Objective possibility and necessity come in many varieties. I could easily type slightly faster than I do; it would be harder but not physically impossible for me to type much faster than I do. As I will use the terms, a proposition is *metaphysically possible* if and only if it has at least one sort of objective possibility. A proposition is *metaphysically necessary* if and only if its negation is not metaphysically possible, that is, if and only if it has every sort of objective necessity. In given circumstances, a proposition is *nomically possible* if and only if it is metaphysically compossible with what, in those circumstances, are the laws of nature (their conjunction is metaphysically possible). A proposition is *nomically necessary* if and only if its negation is not nomically possible—that is, if and only if it is a metaphysically necessary consequence of what, in the circumstances, are the laws of nature. Both metaphysical and nomic modalities are objective. Natural science studies nomic possibility, impossibility, and necessity (among other things). Philosophy, especially metaphysics, studies metaphysical possibility, impossibility, and necessity (among other things). Of course, in everyday speech modal words such as 'can' and 'can't' are typically used to speak about much more restricted kinds of possibility and necessity. Right now, I can reach my keyboard, but I can't reach my bookshelves, even though the laws of physics do not preclude my reaching them. In such examples, the modal words still express objective possibilities or impossibilities, but ones that hold fixed my current circumstances—the position of the chair in which I am sitting, the length and inelasticity of my arms, and so on.

Many linguists use Kratzer's term 'circumstantial modality' in a sense similar to my sense of 'objective modality.' It is particularly appropriate for modalities conditioned on the specific circumstances at hand. It is less appropriate when 'could have' is used to express an equally objective modality that generalizes away from all circumstances. On some reasonable views, however, the universe could have always had fewer dimensions than it actually had. Although 'could' is morphologically the past tense of 'can,' English permits us to recruit past tense forms to express something more purely modal. In saying "The universe could have always had fewer dimensions than it actually had," we acknowledge possibilities that differ from actuality at all past times. We need not be conditioning on any circumstance at all. For our purposes, 'objective' is a more suitable word than 'circumstantial' because it encourages a broader reading that need

not be circumstance-bound, and suggests a relevant analogy with objective probabilities.

The class of objective modalities is plausibly taken to be unified by its closure under various operations. For instance, if being possible$_1$ and being possible$_2$ are two types of objective possibility, then being possibly$_1$ possible$_2$ is also a type of objective possibility, and so is being both possible$_1$ and possible$_2$. There may also be an operation that forms something like the converse of a modal operator, in the sense in which past and future tense operators are something like mutual converses. Given such closure principles, one can show that there is a *strongest* objective necessity operator, the conjunction of all objective necessity operators, which may be identified with *metaphysical necessity*. Thus, metaphysical necessity implies every objective kind of necessity, and dually every objective kind of possibility entails metaphysical possibility. Moreover, one can show from the closure principles that the propositional logic of metaphysical necessity is at least, and presumably exactly, the strong modal system S5, on which the modal status of a proposition as metaphysically necessary, metaphysically contingent, or metaphysically impossible is never itself metaphysically contingent—even though some objective necessity operators satisfy only much weaker modal logics.[2]

The schematic characterization of metaphysical modality as the maximal objective modality leaves many questions about it unanswered. How far does it satisfy Kripke's seminal account (Kripke 1980)? Since the guise under which an object is presented has no bearing on an objective modality, we presumably have the necessity of identity. If metaphysical modality obeys the principles of S5, we can thence derive the necessity of distinctness (Prior 1956). These principles already suffice for some distinctive examples of the necessary *a posteriori*, as we should expect of an objective modality. For instance, it is metaphysically necessary, but not knowable *a priori*, that Socrates is distinct from Plato. But such general structural principles are typically neutral with respect to specific essentialist claims. Nathan Salmon has argued in detail on essentialist grounds that the 4 axiom fails for metaphysical modality; in that sense, something impossible can be possibly possible (Salmon 1982, 1989, 1993). If the essence of an artifact permits small but not large variations in its original constitution, then we should expect the relative possibility of worlds to be non-transitive, for many small differences can add up to a large one. Under the reading of 'metaphysical modality' as meaning the maximal objective modality, Salmon's argument against the 4 axiom must fail. He rejects that reading, and indeed his argument may be sound under some alternative readings of 'metaphysical

[2] Williamson 2016 gives details. There is a plausible argument that, in propositional modal logic, if metaphysical modality obeys at least the principles of S5, then it obeys at most the principles of S5 (Williamson 2013b, 111). For a related but different conception that validates S4 but not S5 for a maximal modality, see Bacon 2016.

modality' as meaning various non-maximal objective modalities with non-transitive accessibility relations. One might even use Kripkean claims as to how different a given object could or could not have been as paradigms with which to explain an alternative sense for 'metaphysical modality.' For present purposes, however, the question is where Salmon's argument goes wrong when read, contrary to his intentions, with respect to the maximal objective modality. One option is to go for a much stricter form of essentialism, on which ordinary claims that an artifact could have had a slightly different original constitution are dismissed as loose talk (Chisholm 1973). Another option is to go for a much less strict form of essentialism, on which ordinary claims that an artifact could not have had a very different original constitution are interpreted as concerning only quite restricted types of possibility (Mackie 2006). There are also intermediate options (Williamson 2013a, 126–143). We need not decide between these options here.

More generally, the conception of metaphysical modality as the maximal objective modality leaves open a wide range of theoretical options. At one extreme, metaphysical modality might involve only a bare minimum of structural constraints, such as the principles of S5 and the necessity of identity and distinctness. At the other extreme, it might involve rich essentialist constraints. It has not even been excluded that metaphysical modality coincides with nomic modality. These questions should be decided by detailed theoretical investigation, not by stipulation. Our present interest is in the whole range of objective modalities, not just in their maximum.

2 Skepticism about Objective Modalities

Objective modalities are supposed to be out there in the world, independently of us. There is a long philosophical tradition of skepticism about such modalities. Its patriarch is of course David Hume. He is usually, and most interestingly, interpreted as calling into question the very idea of objective necessity. On this reading, he denies not merely that we can know that billiard balls *must* rebound from the cushion as they do, but even that we can use the word 'must' to express any idea of objective necessity—as opposed to something psychological in ourselves, such as an expectation that they will so behave. Of course, since correlative types of objective necessity and possibility are interdefinable duals, he is calling into question the idea of objective possibility just as much as the idea of objective necessity. Hume's skepticism targets not only metaphysical modality; his arguments are just as relevant to more restricted objective modalities, such as nomic modality, which may be more appropriate to the motion of billiard balls.

Humean arguments remain surprisingly influential in the philosophy of modality, despite (or even because of) their seeming reliance on *a priori* crudely empiricist assumptions. In particular, it is often still taken for granted that the contents of perception are non-modal. Yet, as I write, I can

see that, from where I sit, I *can* reach the computer screen but not the window. Of course, such modal contents may conceivably be the conclusions of inferences from the contents of one's perception in some stricter sense to be explained, combined with background beliefs about one's body, but such an interpretation is not obviously correct or even especially plausible. A responsible empiricist should at least consider scientific alternatives such as Gibson's theory of perceptual affordances (1979) and its recent successors, on which sense perception has inherently modal contents.[3] After all, from an evolutionary perspective, it would be highly adaptive for perception to present such information about possibilities for action directly to us, rather than leaving us to get there through such time-consuming and troublesome inferences as occur to us. Fast reactions to new perceptual information are often crucial to the success of action.

Recent metaphysics has witnessed a less extreme critique of objective modality, which concedes its intelligibility but denies its fundamentality. Where Quine (1961) dismissed quantified modal logic as incoherent, his student David Lewis (1968) found a way of interpreting it more charitably within a similar broadly Humean framework, using counterpart theory on the assumption that there are many concrete worlds (maximal connected spatiotemporal systems) other than our own. Lewis formulates counterpart theory in a first-order, non-modal language to which Quineans can hardly object, even though they may of course deny that there is more than one world in Lewis's sense. But although his counterpart-theoretic translation presents the quantified modal language as *meaningful*, it also presents it as far from *perspicuous*. The messy complexities of Lewis's translation scheme make the surface forms of quantified modal sentences a misleading guide to their underlying logical relations. From this perspective, it is better to do one's theorizing in the language of counterpart theory itself, free of modal operators, since the latter tend to obscure the deep structure of the metaphysical issues. In his later work, Lewis explicitly did just that, bypassing modal formulations to work directly in the language of counterpart theory (1986). So-called modal realism may just as well be regarded as a form of anti-realism about the modal. Of course, Lewis-style modal realism has never been a majority position. Nevertheless, it has encouraged the tendency not to take modal distinctions at face value, but instead to suspect them of distracting attention from deeper issues.

[3] See Strohminger 2015 for a detailed development of the case for perceptual knowledge of nonactual possibilities; she also documents the widespread reliance in contemporary discussion of the epistemology of modality, following Hume and Kant, on the assumption that there is no such knowledge. Roca-Royes (2017) makes a more empiricist argument for inductive knowledge of nonactual possibilities via their similarity to perceived actualized possibilities: if you have seen cups break, and thereby know that they *can* break, you may infer that a similar unbroken cup can break (though that does less to confront empiricist worries about how we come to understand 'can' in the first place).

A more recent motive for downgrading modality is less Humean than Aristotelian in spirit. Disappointingly, distinctions drawn in modal terms have often turned out to be too coarse-grained to do the metaphysical work initially hoped of them. A prominent case was the attempt to use the notion of supervenience to explain the relation between the mental and physical: no mental difference without a physical difference (Davidson 1970). Despite Davidson's Quinean qualms about modality, supervenience is naturally defined in modal terms (no *possible* mental difference without a physical difference), although making it precise reveals that many subtly inequivalent modal definitions are available. However, on all the most attractive modal definitions, supervenience is not an asymmetric relation, and even one-way supervenience may hold between families of properties that seem to be 'on a level' with each other. Moreover, where supervenience does hold, one wants to know *why* it holds; the suspicion is that the real metaphysical action will be in answering the latter question. Thus, just saying in a precise modal sense that the mental supervenes on the physical, even if true, clarifies the dependence of the mental on the physical much less than had been hoped (see Kim 1993 for discussion). Similarly, following Kripke (1980), it was widely accepted that what it is for a property to be essential to an object can be explained in modal terms; necessarily, if the object exists then it has the property. But Kit Fine (1994) argued persuasively that any such modal definition of essence is too coarse-grained to capture the difference between essential and accidental properties. Such disappointments have contributed to a view of modal distinctions as shallow and inadequate substitutes for metaphysically deeper distinctions concerning essence, grounding, fundamentality, naturalness, constitution, real definition, ontological explanation, or whatever it may be, even if they have some practical utility as a stopgap convenience (Kment 2014 develops a view of this sort).

The focus of such criticism is often specifically *metaphysical* modality, for instance on whether it is metaphysically fundamental and unified or a miscellaneous ragbag of disparate elements (Sider 2011). However, metaphysical modality is just one member of the extended family of objective modalities. Arguments for skepticism about metaphysical modality tend to generalize to other objective modalities, irrespective of the theorist's intentions. For instance, the epistemological challenge "If something is non-actual, how do you know whether it is possible?" arises for *any* non-trivial objective modality, not just for the metaphysical sort—which is not to say that the challenge cannot be met. Likewise, Quine's logical qualms about quantified modal logic do not depend on whether the modal operators are interpreted as metaphysical, nomic, or practical. If Lewis's counterpart theory is used to interpret metaphysical modality, it should also be used to interpret the other objective modalities, which are restrictions of metaphysical modality. Similarly, substituting another objective modality for metaphysical modality just exacerbates the problem of coarse-graining,

for the trouble is that metaphysical necessity comes too cheap, and other forms of objective necessity come even cheaper.

Suppose that *some* non-trivial objective modality, *A*-modality, has the virtues critics deny to metaphysical modality. It is intelligible, but not to be explained in counterpart-theoretic terms, our knowledge of it is reasonable though far from complete, it cuts at a joint, and so on. Perhaps we should not *identify* metaphysical modality with *A*-modality, because some more general objective modality has all those virtues too. Still, given the virtues of *A*-modality, standard critiques of metaphysical modality are clearly missing something of crucial importance. Those who seek to disarm metaphysical modality had better disarm the whole family of objective modalities, lest other family members exact their revenge.

The point is not that *A*-modality may be able to cut finer than metaphysical modality, for by hypothesis *A*-modality is an objective modality, and on the view developed in this paper metaphysical modality is the maximal objective modality; therefore, if it is *A*-possible for two things to come apart, then it is metaphysically possible for them to come apart. Rather, the point is that if *A*-modality is metaphysically significant despite its failure to cut finely in the ways the critics accuse metaphysical modality of failing to cut, then the failure of metaphysical modality to cut finely in those ways does not show it to be metaphysically insignificant.

One response is that *nomic* modality is a non-trivial objective modality that does not stand or fall with metaphysical modality, because it can be independently explained in terms of natural science: to be nomically possible is to be consistent with the laws of nature, and natural science is our best source of knowledge about those laws.

What does 'consistent' mean there? Suppose that it means *logically consistent*. But the sentence "Hesperus $\neq$ Phosphorus" is logically consistent with the laws of nature, for their formulation involves nothing as parochial as the names 'Hesperus' and 'Phosphorus,' and the sentence "Hesperus = Phosphorus" by itself is no truth of logic. For the same reason, the sentence "Hesperus is a quark" is also logically consistent with the laws of nature. The proposal is supposed to characterize a kind of *modality*, which should classify propositions rather than sentences, but if it is to preserve the difference in logical consistency between the sentences "Hesperus $\neq$ Phosphorus" and "Hesperus $\neq$ Hesperus" it will have to distinguish the proposition that Hesperus is distinct from Phosphorus from the proposition that Hesperus is distinct from Hesperus, presumably by treating "the proposition that . . ." as something like a quotational context. On such a view, it is nomically possible that Hesperus $\neq$ Phosphorus but not nomically possible that Hesperus $\neq$ Hesperus. It is also nomically possible that Hesperus is a quark. That is not an attractive view of nomic possibility. Indeed, it is not even a consistent view, since nomic modality is by hypothesis an objective modality. For one mark of objective modality is that it does not block the substitution of co-referring names. Thus if it is nomically possible that Hesperus $\neq$

Phosphorus, it is also nomically possible that Hesperus ≠ Hesperus, in which case the view requires the proposition that Hesperus ≠ Hesperus be logically consistent with the laws of nature, which it is not, because it is not even logically consistent with itself. The difference between the names 'Hesperus' and 'Phosphorus' is a difference in our representations that corresponds to no difference in the states of affairs represented or their objective modal status. The same problems arise if one appeals to 'conceptual consistency' (whatever that is) instead of logical consistency. To avoid the problems, in defining nomic possibility one would have to conjoin the laws of nature with all true claims of identity and distinctness, such as "Hesperus = Phosphorus," and true claims of kind membership and non-membership, such as "Hesperus is a planet" and "Hesperus is not a quark." But those are just the sorts of move philosophers make in trying to reductively define metaphysical modality itself. It is an illusion that one can define a nomic objective modality without running into the issues that beset metaphysical modality. One might as well admit that nomic possibility is metaphysical compossibility with the laws of nature. If metaphysical modality is in trouble, so is nomic modality. Like other objective modalities, it depends on metaphysical modality. Indeed, for many purposes, though presumably not all, we may even be able to work with the hypothesis that nomic modality coincides with metaphysical modality.

The skeptic may respond: if the other objective modalities depend on metaphysical modality, so much the worse for them, and in particular for nomic modality. Even though the appeal to natural science is necessary for nomic modality, it is not sufficient. According to such a skeptic, natural science can in principle be done without reliance on objective modalities; the science does not vindicate their specifically modal aspect. In a scientistic climate, such an assumed lack of connection to natural science makes the objective modalities look suspiciously ill-grounded.

It is notable how minor a role natural science plays in current discussion of the epistemology of modality. The main emphasis is on folk methods of knowing whether something is possible, perhaps by imaginative means, described in one way or another. One would expect such folk methods to be geared primarily to quite restricted forms of practical modality, although philosophers usually want to discuss knowledge of metaphysical modality. One might get the impression that philosophers have taken some practically convenient everyday ways of thinking ('can'), drastically generalized them ('metaphysically possible'), perhaps far beyond their domains of reliability, and on those tenuous foundations erected a shaky castle of philosophical theory. Implicit in this picture is that science itself has no essential objective modal aspect, so its track record of success offers no support to the enterprise of objective modal theorizing. For instance, Ted Sider (2011) claims that "modality is unneeded for the most fundamental inquiries" (267).

There is no need to disparage folk methods of gaining knowledge about various types of objective possibility and necessity, perhaps including metaphysical possibility and necessity. Such methods are easy to underestimate, but they are not the focus of this paper; rather, its concern will be with more scientific methods of learning about objective modalities. Of course, logic is itself a science—in some ways the most rigorous science of all—and the study of quantified modal logic with respect to objective interpretations of the modal operators is a branch of that science. Arguably, it is best pursued by abductive methods of theory choice similar to those used in the natural sciences (Williamson 2013b), but this paper does not take that view of modal logic for granted. Instead, it asks to what extent there is an implicit (or explicit) objective modal dimension to what are ordinarily counted as natural sciences, and to ordinary mathematics as applied in those sciences.

The mere definition of 'nomic possibility' as compossibility with the laws of nature poses no threat to the picture of natural science itself as non-modal, since it does not imply that the idea of nomic possibility plays any essential role in scientific attempts to identify the laws of nature. For all that the definition shows, modal ideas might be merely epiphenomenal in the scientific process. Likewise with the deduction of nomic possibility from actuality: if natural science discovers that there are black holes, we can of course deduce that it is nomically possible for there to be black holes, but that offers natural scientific help only where it is least needed, since the hard question is how far the possible extends *beyond* the actual.

We could go through numerous articles in journals of natural science and list all the places where modal expressions are used, in plainly objective senses, but we are unlikely to achieve much just by doing so. For such articles are written in mostly natural language, and one can expect authors often to fall into such everyday ways of expressing themselves, even where they are not strictly needed. How might objective modality play a more essential role in natural science?

An apparent reason for pessimism is the increasing extent to which, as natural scientific theories become more rigorous, their core is expressed in equations or other mathematical formulas. For the language of mathematics is non-modal. It does not contain symbols like $\Box$ and $\Diamond$, at least not to mean necessity and possibility. Of course, the absence of modal expressions *within* the formulas does not preclude us from ascribing nomic or metaphysical necessity to them from the outside. If "5 + 7 = 12" and "$E = mc^2$" are purely non-modal statements, we may still affirm that it is metaphysically necessary that 5 + 7 = 12 and nomically necessary that $E = mc^2$. But the danger is that such modal claims are merely philosophers' exogenous honorific glosses, functionless within the science itself. It is as if a philosopher went round sticking gold labels on his favorite machines, reading "This machine has been approved by a qualified metaphysician." The label may look good, but it makes no difference to the working of

the machine. More specifically, if the modal glosses are merely external to the science, then they draw no abductive support from the explanatory successes of the science. Can we find cases where instead the modal glosses reflect some endogenous need of the science? The next section starts to come to grips with that question.

3 Laws Support Counterfactuals

One obvious starting-point is the near-platitude in the philosophy of science that laws support counterfactuals. If it is a law that all *F*s are *G*s, then if there had been an *F*, there would have been a *G*. Even if the universal generalization itself contains no modal element, in claiming that it is a law we endorse its application to (at least some) counterfactual circumstances. Surely we want to use our scientific theories, including our mathematical theories, in reasoning about how things could have been, as well as about how they are. In such reasoning, we engage a specifically objective modal dimension. For those purposes, the core theory itself need not be cast in modal terms. It is enough that sometimes our legitimate applications of it assign it a modal status. What's the problem?

Not all reasoning from a false hypothesis is counterfactual in the sense relevant to objective modality. To use a standard example, the uncontentious truth "If Oswald didn't shoot Kennedy, someone else did" is an ordinary indicative conditional, even though Oswald did in fact shoot Kennedy. It is uncontentious because, for sure, *someone* shot Kennedy, so if it wasn't Oswald, it was someone else. It is not equivalent to the so-called subjunctive conditional "If Oswald hadn't shot Kennedy, someone else would have," which suggests another back-up conspirator lying in wait. Unlike the subjunctive conditional, the indicative conditional does not involve an objective modality. The indicative conditional, but not the subjunctive conditional, can be reasonably inferred from the non-modal statement "Someone shot Kennedy." Indicative and subjunctive conditionals interact differently with modally rigidifying devices, such as 'actually' and 'in this world' (Williamson 2006). "If Oswald didn't shoot Kennedy, someone else did" is equivalent to "If Oswald didn't shoot Kennedy, someone else did in this world." By contrast, "If Oswald hadn't shot Kennedy, someone else would have" is not equivalent to "If Oswald hadn't shot Kennedy, someone else would have in this world," since the former is true and the latter false in the scenario where Oswald's shot pre-empts an efficient backup assassin. In that respect, subjunctive conditionals pattern like objective modals while indicative modals do not. Although the epistemic modal sentences "Oswald may have missed" and "Oswald may have missed in this world" are more or less equivalent, the objective modal sentences "Oswald could have missed" and "Oswald could have missed in this world" are not; indeed, the former is true and the latter false (where in both cases

"in this world" is read as within the scope of the modal verb).[4] In considering applications of scientific theories (including mathematical theories) to hypothetical situations, we must be careful about whether they really require subjunctive conditionals rather than indicative ones, even when it is natural for us to articulate them in terms of subjunctive conditionals.

Imagine that we are assessing a plan *A* for building a bridge. We ask "What would happen if we were to build the bridge according to plan *A*?" We apply our relevant theories, and come to the conclusion "If we were to build the bridge according to plan *A*, it would fall down." Consequently, we decide not to build the bridge according to plan *A* (so the antecedent of the conditional is false). We reasoned with subjunctive conditionals, and it was quite natural to do so. But there was no real need to do so. We could just as well have reasoned with indicative conditionals, asking "What will happen if we build the bridge according to plan *A*?" applying our relevant theories as before, and concluding "If we build the bridge according to plan *A*, it will fall down." If we know that indicative conditional in those circumstances, we have reason enough not to build the bridge according to plan *A*.[5] It can be natural to articulate an application of a theory in subjunctive terms even when there is no real need to do so. We might wonder for a moment whether an objective modal dimension, by contrast with an epistemic modal dimension, is ever really needed in practical applications.

Consider learning from mistakes. We see a bridge fall down. We ask the subjunctive conditional question "What would have happened if the bridge had been built according to plan *B*?" Then we apply our relevant theories, and come to the subjunctive conditional conclusion "If the bridge had been built according to plan *B*, it would not have fallen down." Consequently, we do better ourselves next time we have to build a bridge. In this case, indicative conditionals will not do just as well. We do not assert "If the bridge was built according to plan *B*, it did not fall down," for we know for sure that it did fall down, whether or not it was built according to plan *B*. Even if we treat the indicative conditional as truth-functional, true simply because its antecedent is false (we have concluded that the bridge was not built according to plan *B*), we should regard it as too conversationally misleading to assert. It is the subjunctive conditional that carries the relevant information. For such applications, we need an objective modal dimension.

Causal hypotheses are also far more strongly connected to subjunctive conditionals than to indicative conditionals, although it may be overly

[4] It is plausible but not trivial that 'actually' and 'in this world' can be read as rigidifying objective modal operators that form metaphysically non-contingent propositions materially equivalent to the proposition to which they are applied. For instance, "In this world . . ." may be equivalent to "It is metaphysically necessary that if this world obtains then . . . ."

[5] The difference between indicative and subjunctive readings is hard to hear for such future tense sentences. That does not undermine the point in the text, since their utility does not depend on a distinctively subjunctive reading.

optimistic to expect strictly necessary and sufficient conditions for causal hypotheses in terms of subjunctive conditionals, or strictly necessary and sufficient conditions for subjunctive conditionals in terms of causal hypotheses.

Thus, even if we start with an austerely formulated scientific or mathematical theory, free of modal and causal vocabulary, applying the theory for practical purposes or to reach causal conclusions often depends on its supporting subjunctive conditionals. When those applications are successful, part of what their success abductively supports are those subjunctive conditionals. It can be argued in detail that if a scientific theory supports subjunctive conditionals, it also supports ascriptions of nomic necessity (Williamson 2016). In brief, if we want to apply a scientific theory freely in the scope of subjunctive reasoning about nomic possibilities, the theory had better be at least nomically necessary, even if the content of the theory itself is purely non-modal. Furthermore, if such modal applications have a track record of success, it provides abductive confirmation for the relevant claims of nomic necessity. It is a mistake to picture those objective modal claims as supported by nothing more than folk habits of thought and metaphysical speculation. Nevertheless, that point does not eliminate the suspicion that conceptualizing matters in such supposedly objective modal terms somehow misleads us about the underlying fundamental joints in nature. Thinking in terms of natural language subjunctive conditionals may look prescientific. These concerns may be fuelled by the absence of objective modal constructions from the language in which many scientific theories are formulated. Such suspicions can best be answered by considering cases where the content of the scientific theory itself is objectively modal. That is the task for the rest of the paper.

4 Objective Probabilities

Talk of probabilities is, of course, widespread in the natural sciences. Although casual uses of the word 'probably' may merely express caution, explicit quantification of probabilities—for instance, in the interpretation of statistics—presupposes some form of modality, for any probability distribution is defined over a probability space of mutually exclusive, jointly exhaustive 'possibilities'; in any given circumstances, all but one of them is counterfactual. Probabilities are assigned to all members of a field of 'events'—that is, subsets of the set of all those possibilities. The possibilities behave like possible worlds, and the events behave like coarse-grained propositions, sets of possible worlds. The probability of a proposition is a measure of its closeness to necessity. If the number of events is finite, all nonempty propositions may be assigned nonzero probability, in which case probability 1 corresponds to necessity, because it is equivalent to 'truth everywhere' in the space, and nonzero probability corresponds to possibility, because it is equivalent to 'truth somewhere' in the space. If the number

of events is infinite, there are technical obstacles to assigning all nonempty propositions nonzero probability; probability 1 is equivalent only to 'truth almost everywhere' in the space. But even in the latter case, probability 1 and nonzero probability still behave like dual modal operators. Moreover, necessity and possibility can still be defined in a natural way over the probability space as 'truth everywhere' and 'truth somewhere' respectively. A one-way connection still holds between probability and possibility; whatever has nonzero probability is possible in the corresponding sense, even though the converse fails. Moreover, probabilistic distinctions resemble modal distinctions in being coarse-grained; just as truth-functionally equivalent formulas are necessarily equivalent, they also have the same probability, as a consequence of the standard axioms for probability.

For present purposes, however, not any old probabilities will do. Only *objective* probabilities are appropriately related to objective possibilities. Often the probabilities discussed in science are epistemic, dependent on an evidence base, and so not suitable here. Subjective probabilities (credences, degrees of rational belief) help still less. Indeed, not even all objective probabilities are interesting for our purposes, since some of them are in effect distributions only over sets of actual cases. In particular, we are not concerned with probabilities understood as actual frequencies, even though they are objective at least in being agent-independent. But frequentist interpretations of probability are in any case unpromising, because actual frequencies may happen to be utterly wayward, in principle even over a very long run. A fair coin can come up heads any number of times in succession. Frequencies are better understood as good *evidence* for underlying probabilities that explain, and so are not to be identified with, the frequencies.

The most familiar genuine objective probabilities are *chances*. In a deterministic system, chances are all 1 or 0. But if the system is indeterministic, they may, more interestingly, be intermediate numbers. The most celebrated example of indeterminism and intermediate chances in science is of course in quantum mechanics, under some interpretations. It is widely accepted that the probabilities in the formulation of quantum mechanics are not merely epistemic or subjective. However, given the notorious difficulties of interpreting quantum mechanics, we shall leave discussion of its probabilities to the experts.

Significantly, the very distinction between deterministic and indeterministic systems itself involves objective modality. Suppose that you are given the entire history of a system, past, present, and future, all described in purely non-modal terms, and that the history contains no recurrences; the system is in each maximally specific state at most once. There is no way of reading off from the history whether the system is deterministic or indeterministic. That depends on whether there are two *possible* total histories of the system that coincide at one time and diverge at another. The type of possibility at issue is nomic and objective. Chance is a measure of closeness to a

timebound sort of objective necessity, such as nomic necessity conditioned on the circumstances at the time.

Non-trivial objective probabilities may also arise for deterministic systems. For scientists may explain some general features of the system's actual total history by showing them to be *typical* of its possible histories. That is, it is highly probable that the system will have a total history with those features. The relevant probabilities here are not chances given the state of the system at a time, but rather something like probabilities over initial conditions. For the explanation to work properly, those probabilities should be objective. If we were merely told that it would be rational for someone in a particular evidential situation to be confident that the system would have a total history with the features at issue, we should be unsatisfied, because such a hypothetical agent is quite extraneous to what was to be explained. A better explanation would strip out the irrelevant material about the agent and isolate the relevant facts about the system itself that the non-objective 'explanation' was clumsily attempting to communicate. An example of such an explanation of the general behavior of a deterministic system in terms of objective probabilities over its initial conditions is the derivation of standard thermodynamic principles from classical statistical mechanics (see Loewer 2001 and Maudlin 2007 for discussion).

For illustrative purposes, a toy example will suffice instead. Suppose that a coin was tossed 1000 times. It came up heads approximately 500 times (the explanandum). Why? A potential explanation is that the coin was fair and the tosses mutually independent (the explanans). Once the relevant calculations are made, the explanans gives a reasonable explanation of the explanandum. It is a piece of proto-science. Probability enters the explanation in at least two ways. First, the explanans itself is implicitly probabilistic; the coin is said to be fair in the sense that the probability of heads on a given toss is 1/2, and the tosses are said to be mutually independent in the sense that the unconditional probability of an outcome of a given toss equals its probability conditional on given outcomes of other tosses. Second, the connection between the explanans and the explanandum is also probabilistic, since the explanans does not entail the explanandum—the explanans is consistent with the coin's coming up heads every one of the 1000 times—but instead only makes the explanandum probable (in the same sense of 'probable'). The relevant probabilities are not subjective or epistemic, since the degrees of belief or evidential situation of an actual or ideal agent played no relevant role in the event to be explained. They are quite extraneous to the explanandum and should not figure in the explanans.[6] Nor are the relevant probabilities frequentist. For consider any

[6] If we used evidential or subjective probabilities, the putative explanation would at best show that the explanandum "was to be expected." But to show that an outcome was to be expected is not to explain why it occurred, in the relevant broadly causal sense. Whether it was to be expected depends on the guise under which it was presented in a way that its causal explanation does not.

given toss in the long run over which such frequencies would have to be calculated. If the toss is one of the 1000 in the explanandum, that would make for circularity in the explanation, but if the toss is not one of the 1000, then it played no role in bringing about the explanandum. Either way, it should be excluded. The example is best understood as involving a reasonable proto-scientific explanation in terms of objective probabilities.

The example does not require the physics underlying coin tossing to be indeterministic. Instead, each cell of the macroscopic probability space may correspond to one equivalence class of a coarse-grained macroscopic partition of possible microscopic deterministic histories that differ from each other in the past and present as well as on the future; microscopically different ways of tossing the two coins lead deterministically to macroscopically different outcomes. Such possibilities are just as objective as indeterministic chances; no 'initial conditions' were nomically necessary. We should not suppose that an explanation in terms of the detailed microscopic histories of the actual tosses would in principle be better. For an explanation of the latter sort involves a drastic loss of generality: its microscopic explanans would obtain in only a tiny fraction of the cases in which the explanandum (as characterized above) would obtain. To capture the generality of the explanandum, we need the generality of the macroscopic objectively probabilistic explanans. A proper microscopic explanation would involve objective probabilities over different microscopic possibilities that realize the initial conditions of the coin tossing.

5 State Spaces

Probability is far from the only form in which objective modalities become the object of natural scientific inquiry. It is standard practice to study a physical system by analyzing its state space or phase state, the abstract space of its *possible* states. The system may be as large as the universe or as small as a few interacting fundamental particles. The type of possibility is objective, more or less nomic. The states are maximally specific.

State spaces have played a philosophically significant role in various connections. For example, in his critique of Hartry Field's (1980) nominalizing program, David Malament (1982, 533) objects that if Field's method of nominalization is applied to various theories of mechanics, its effect is to replace quantification over abstract objects by quantification over the "'possible dynamical states' (of particular physical systems)," to which, he argues, a nominalist is not entitled. Aidan Lyon and Mark Colyvan (2008) have taken the latter objection further, arguing that attempts to nominalize standard phase-space theories in physics would result in a loss of explanatory power; as they explain, "phase spaces are spaces of possible, but mostly non-actual, initial conditions" (227). In none of these cases are the possibilities subjective or epistemic; they are aspects of the physical domain under study, not of any real or ideal physicist's state of knowledge

or belief. Rather, they are in some sense objective possibilities. For present purposes, our concern is not with the prospects for nominalization.[7] Rather, it is with the objectively modal dimension of the physics.

To develop the point, we may consider for a case study the theory of dynamical systems (Strogatz 2001). In itself it is a mathematical theory, but it has intended applications in physics, chemistry, biology, and engineering, for instance to a pendulum, the solar system, the population growth or decline of predator and prey species, the weather, and so on. As the last case suggests, it is a standard framework for the study of chaotic systems.

Mathematically, a dynamical system is built on a set S of what are informally understood as the *instantaneous states* of the target system; they are maximally specific in relevant respects, mutually exclusive, and jointly exhaustive. Some geometrical or topological structure is defined on the set of states. The dynamics are given by a family of functions, indexed by what are interpreted as lengths of time—positive (forward in time) or negative (backward). These functions map states to states. Informally, given an input state, such a function outputs the state the system is in the indexed length of time after it was in the input state. The functions are constrained to fit together into a consistent dynamics. The system is assumed to be deterministic in both past and future directions; thus, given its state at any one time, the dynamics fixes its state at any length of time from then. Time may be treated as either continuous or discrete.

The mathematical theory of dynamical systems is just a branch of regular, non-modal mathematics. However, most *intended applications* of that mathematical theory are modal, in the sense that S is interpreted as the set of *possible* states of the target dynamical system—not, for instance, just the set of actual past, present, and future states of the system. To be more precise, given a dynamical system, let an *orbit* be the set of states which the system goes through at some time or other in a single history consistent with the dynamics. If some orbit contains all states, it is the only orbit. But, typically, no orbit contains all states. Thus some of the states are mutually incompossible, given the dynamics, in the strong sense that if the system is *ever* in one of them, it is *never* in the others. The states are *possible states*, not all of which are ever actualized. It would be foolish to try to eliminate all the counterfactual states in the system by cutting it down to just its actual orbit, because that would typically destroy the geometrical or topological structure defined over the states: that structure is crucial to the explanatory power of the theory of dynamical systems. The point is to study the dynamical system of possible states as a whole, exploiting that mathematical structure.

As before, the sort of possibility at issue is not subjective or epistemic. It depends on the nature of the physical system under study, not on the

[7] One may question the assumption that merely possible states of a physical system are abstract objects (Malament 1982, 533; Lyon and Colyvan 2008, 233). On the approach of Williamson 2013b, 7, their non-concreteness does not make them abstract.

psychological or epistemic states of the theorist who studies it, or of anyone else, real or ideal. It is some sort of objective possibility, usually nomic rather than metaphysical, perhaps even more restricted. Of course, dynamical systems are mathematical models of complex and often messy natural structures, and as such are likely to involve some degree of simplification, idealization, and approximation. But that is just the normal case with natural science. It does not mean that they have nothing to tell us about reality.

The possible states are clearly quite like *possible worlds*; however, since states are instantaneous, they are even more like ordered pairs of a world and a time, such as one evaluates formulas at in some formal theories of semantics for languages with both modal and temporal operators. But not even that comparison is perfectly apt, for nothing in a state specifies *when* the system is in it; indeed, unlike world-time points, states may be *repeatable*. When a dynamical system exhibits cyclic behavior, it will be infinitely many times in each state that it is ever in (Nietzsche's eternal recurrence). One might try saying that the states are qualitative in a way that world-time points are not, but even that claim may be misleading, since the mathematical structure on states may require treating the result of spatially rotating, reflecting, or translating a given state as a different state, even though they are qualitatively indistinguishable. Thus, states cannot be perfectly assimilated into the framework of possible world semantics. Nevertheless, they quite clearly have an objective modal aspect, as well as a qualitative-temporal one.

We can make the modal aspect of dynamical systems explicit by treating them as models over which we can evaluate formulas of a modal language. This can be done in a very smooth and natural way, without applying any Procrustean methods. By their structure, dynamical systems ask to be so treated. More specifically, given any dynamical system, one can evaluate formulas as true or false at a state, just as formulas of an ordinary modal language are evaluated as true or false at a world in Kripke-style models for modal logic. Such sets play the role of propositions in the model, just as sets of worlds play the role of propositions in Kripke models. The language has the usual truth-functors, which behave as expected. It has modal operators, treated like quantifiers ranging over all states of the system (in this language, they need not express metaphysical modalities).[8] It also has temporal operators such as 'at some future state,' 'at every future state,' 'at some past state,' and 'at every past state,' defined in terms of the dynamics of the system. There are also propositional variables in sentence position and propositional quantifiers, allowing one to generalize over all 'propositions'—that is, sets of states. Further sentence operators express relevant mathematical aspects of the dynamical system. Natural semantic

[8] They are unrestricted in the sense of ranging over all states of the system; they are typically not equivalent to metaphysical necessity and metaphysical possibility.

clauses for all these operators enable one to treat each dynamical system as a model for the modal-temporal language (see Williamson 2016 for details).

A formula is *valid* on such a model if and only if it is true in the model at every state on every assignment of values to the propositional variables. Any class of dynamical systems defines a modal-temporal logic whose theorems are the formulas valid on all models corresponding to systems in the class. The underlying non-modal propositional logic is always classical; all truth-functional tautologies are valid, and modus ponens preserves validity. The unrestricted modal operators obey all the principles of the modal system S5. Consequently, if something is ever possible, it is always possible.[9] We also have standard principles of tense logic for linear time with no first or last moment, which derive from the additive structure of the real numbers or the integers. Other axioms, such as those corresponding to the density or discreteness of the time order, are validated if we restrict validity to continuous or to discrete models. The quantifiers obey the standard principles for propositional quantifiers in a modal setting, including for each state the existence of a proposition true at exactly that state. The language can express such ideas as Nietzsche's eternal recurrence. It can also express key ideas in the theory of chaos, such as the idea of an *attractor* and of a *basin of attraction*, which help to explain the long-term qualitative behavior of dynamical systems (see Strogatz 2001, 324).

From this perspective, the non-modal nature of the mathematics that constitutes dynamical systems theory looks no more metaphysically significant than the non-modal nature of the mathematics that constitutes possible worlds model theory. In both cases, the modal connection is made by the intended applications of the mathematics. It is very convenient to reason in the non-modal language of mathematics, but in many applications we implicitly or explicitly characterize the entities we are reasoning about in modal terms. In metaphysics we may reason about modality by quantifying over possible worlds; in natural science we reason about modality by quantifying over possible states of a system. In the former case, the relevant modality is metaphysical; in the latter, it is more like nomic, but in both those cases it is objective. Natural science studies the structure of spaces of objective possibilities just as much as metaphysics does.

6 Metaphysical Versus Other Objective Modalities

Appeals to objective modal aspects of natural science seem to have this limitation: they concern at best some form of nomic modality, but not metaphysical modality. How much light do the arguments of sections 3 through 6 cast on metaphysical modality?

[9] This 'diamonds are forever' principle is reminiscent of, but not equivalent to, the principle defended by Dorr and Goodman Forthcoming; the latter concerns metaphysical possibility and a more standard reading of the tense operators.

The gap between nomic and metaphysical may be narrower than is usually thought. Following Saul Kripke (1980), Alexander Bird (2007) has argued in detail that laws of nature may be metaphysically necessary. If what it is to be an *F* involves being a *G*, then it is metaphysically, not just nomically, necessary that all *F*s are *G*s. It is a good question how far such arguments can be taken: Could not different laws of nature have obtained? In any case, the total assimilation of nomic modality to metaphysical modality is not only rather implausible; it is not even relevant to all the cases discussed above. For the possibilities in a probability space or state space may not even exhaust all nomic possibilities, let alone all metaphysical ones. They may cover just the possible states of a highly contingent system, such as the tossing of coins or the weather on earth. Still, we may assume that in a typical case they *are* nomic possibilities, even if they are not all of the nomic possibilities.

A simple point is that nomic possibility entails metaphysical possibility, the most general type of objective probability. Thus if science shows something to be nomically possible, it thereby shows it to be metaphysically possible too. The nomic possibility of various states is built into applications of probability spaces and state spaces. Curtailing the state space typically disrupts its mathematical structure and thereby reduces the explanatory power of the theory. Of course, someone might challenge the entailment from nomic possibility to metaphysical possibility. We saw in section 2 that if nomic possibility is just logical consistency with the laws of nature, it does not entail metaphysical possibility. However, that account of nomic possibility fared very badly. If instead nomic possibility is metaphysical compossibility with the laws of nature, then it trivially entails metaphysical possibility, for a metaphysical impossibility is not metaphysically compossible with anything. There is no good reason to deny the entailment from nomic possibility to metaphysical possibility.

Although no attempt will be made here to argue in general from nomic necessity to metaphysical necessity, often the main challenge to a claim that something is metaphysical necessary is also by implication a challenge even to the claim that it is nomically necessary. In such cases, the main dialectical action is within the realm of nomic possibility. The distinction between nomic and metaphysical necessity, though granted, sometimes makes less difference than might have been expected to the modal upshot of natural science.

Those considerations combine naturally with the more general observations in sections 1 through 2 of how the objective modalities tend to stand or fall together. It is implausible to treat the objective modal dimension of natural science as merely an artifact of folk cognitive architecture or overheated metaphysical speculation. We have no good evidence that it is a proxy for something else.

We can draw a moral about the epistemology of modality. It is not autonomous. Our natural scientific knowledge of objective modality is too

tightly integrated with the rest of our scientific knowledge to permit that. Unsurprisingly, the abductive methodology of natural science plays a major role in the epistemology of modality. That conclusion also has consequences for skeptical and deflationary theories about the metaphysics of modality, which are typically motivated by assumptions about the epistemology of modality that do not withstand scrutiny.[10]

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[10] A version of this paper was given as the 2016 Wade Lecture at Saint Louis University; I thank the audience and two referees for *Res Philosophica* for helpful comments. The paper is an informal presentation of ideas developed in more technical detail in Williamson 2016

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